Effects of factors

Main effects: Expected average response when the factor is on its high level - Expected average response when the factor is on its low level, i.e $E(\bar{Y}_H - \bar{Y}_L)$

Estimate $\bar{y}_H - \bar{y}_L$

For temperature: $\hat{A} = \frac{79+75}{2} - \frac{61+67}{2} = \underline{13}$

Notice: $b_A = (\underline{X}_A' \underline{X}_A)^{-1} X_A' \underline{y} = \frac{1}{4}(-67+79-61+75) = \underline{6.5}$

For Carbon: $\hat{B} = \frac{61+75}{2} - \frac{79+67}{2} = \underline{-5}$

and $b_B = (\underline{X}_B' \underline{X}_B)^{-1} \underline{X}_B' \underline{y} = \frac{1}{4}(-67-79+61+75) = \underline{-2.5}$

Interaction between two factors: Half the main effect of a factor when the other is on its high level - half the main effect of a factor when the other is on its low level.

$\hat{AB} = \frac{75-61}{2} - \frac{79-67}{2} = 1$

$b_{AB} = (\underline{X}_{AB}' \underline{X}_{AB})^{-1} \underline{X}_{AB}' \underline{y} = \frac{1}{4}(67-79-61-75) = 0.5$

Signs for computing effects

Temp	Carbon	Temp × Carbon
−	−	+
+	−	−
−	+	−
+	+	+

Other level codes

A	B	
−	−	1
+	−	a
−	+	b
+	+	ab

2^3 eksperiment

A	B	C
Temperature	Carbon	Temperature in
− +	− +	− +

A	B	C	AB	AC	BC	ABC	level code	y
−	−	−	+	+	+	−	1	67
+	−	−	−	−	+	+	a	79
−	+	−	−	+	−	+	b	61
+	+	−	+	−	−	−	ab	75
−	−	+	+	−	−	+	c	59
+	−	+	−	+	−	−	ac	90
−	+	+	−	−	+	−	bc	52
+	+	+	+	+	+	+	abc	87

$$\hat{A} = \frac{79+75+90+87}{4} - \frac{67+61+59+52}{4} = 23$$

$$\hat{B} = \frac{61+75+52+87}{4} - \frac{67+79+52+90}{4} = -5$$

$$\hat{C} = \frac{59+90+52+87}{4} - \frac{67+79+61+75}{4} = 1.5$$

$$\hat{AB} = \frac{67+75+59+87}{4} - \frac{79+61+90+52}{4} = 1.5$$

$$\hat{AC} = \frac{67+61+90+87}{4} - \frac{79+75+59+52}{4} = 10$$

$$\hat{BC} = \frac{67+79+52+87}{4} - \frac{61+75+59+90}{4} = 0$$

$$\hat{ABC} = \frac{80+52+54+72}{4} - \frac{83+45+60+68}{4} = 0.5.$$

<u>Evaluation of significant effects in unreplicated experiments</u>

$$\hat{Effect} = \frac{\sum_{i=1}^{m} \delta_i Y_i}{\frac{m}{2}}, \quad m \text{ is the number of observations and } \delta_i \text{ is either } 1 \text{ or } -1$$

$$\sigma^2_{effect} = \sum_{i=1}^{m} \frac{\delta_i^2 Var(Y_i)}{\frac{m^2}{4}} = 4\frac{\sum_{i=1}^{m} \sigma^2}{m^2} = \frac{4m\sigma^2}{m^2} = \frac{4\sigma^2}{m}$$

Assume all effects are zero and that Y_i are normally distributed with the same variance, $i=1,2,\dots,m$

Then the estimators of the effects are $N(0, \frac{4\sigma^2}{m})$

In a normal probability plot the estimated effects should all be lying on a straight line. Those who fall off can be considered to be significant.